Appearance of fractional charge in the noise of non-chiral Luttinger liquids

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The current noise of a voltage biased interacting quantum wire adiabatically connected to metallic leads is computed in presence of an impurity in the wire. We find that in the weak backscattering limit the Fano factor characterizing the ratio between noise and backscattered current crucially depends on the noise frequency ω relative to the ballistic frequency v_F/gL, where v_F is the Fermi velocity, g the Luttinger liquid interaction parameter, and L the length of the wire. In contrast to chiral Luttinger liquids the noise is not only due to the Poissonian backscattering of fractionally charged quasiparticles at the impurity, but also depends on Andreev-type reflections at the contacts, so that the frequency dependence of the noise needs to be analyzed to extract the fractional charge $e^* = eg$ of the bulk excitations.

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Shot noise measurements are a powerful tool to observe the charge of elementary excitations of interacting electron systems. This is due to the fact that in the Poissonian limit of uncorrelated backscattering of quasiparticles from a weak impurity, the low frequency current noise is directly proportional to the backscattered charge [1]. This property turns out to be particularly useful in probing the fractional charge of excitations in one-dimensional (1D) electronic systems, where correlation effects destroy the Landau quasiparticle picture and give rise to collective excitations, which in general obey unconventional statistics, and which have a charge different from the charge e of an electron [2]. In particular, for fractional quantum Hall (FQH) edge state devices, which at filling fraction $\nu = 1/m$ (m odd integer) are usually described by the *chiral* Luttinger liquid (LL) model, it has been predicted that shot noise should allow for an observation of the fractional charge $e^* = e\nu$ of backscattered Laughlin quasiparticles [3]. Indeed, measurements at $\nu = 1/3$ by two groups [4, 5] have essentially confirmed this picture. The question arises whether similar results can be expected also for *non-chiral* LLs, which are believed to be realized in carbon nanotubes [6] and single channel semiconductor quantum wires [7]. Although a non-chiral LL can be modelled through the very same formalism as a pair of chiral LLs, some important differences between these two kinds of LL systems have to be emphasized. In particular, in chiral LL devices right- and left-moving charge excitations are spatially separated, so that their chemical potentials can be independently tuned in a multi-terminal Hall bar geometry. In contrast, in non-chiral LL systems, right- and left-movers are confined to the same channel, and it is only possible to control the chemical potentials of the Fermi liquid reservoirs attached to the 1D wire. This in turn affects the chemical potentials of the right- and left-moving charge excitations in a non-trivial way depending on the interaction strength, and implies crucial differences between chiral and non-chiral LLs, for instance, the conductance in the former case depends on the LL parameter $g = \nu$ [8], while in the latter case it is independent of g [9, 10, 11]. Hence, the predictions on shot noise properties of FQH systems are not straightforwardly generalizable to the case of non-chiral LLs, which therefore deserve a specific investigation. Previous theoretical calculations of the shot noise of non-chiral LL systems have shown that, even in the weak backscattering limit, the zero frequency noise of a finite-size non-chiral LL does not contain any information about the fractional charge backscattered off an impurity [12, 13], but is rather proportional to the charge of an electron. This result, as well as the above mentioned interaction independent DC conductance, prevents easy access to the interaction parameter g.

On the other hand, a quantum wire behaves as a Andreev-type resonator for an incident electron, which gets transmitted as series of current spikes [9]. The reflections of charge excitations at both contacts are called Andreev-type reflections because they are momentum conserving as ordinary Andreev reflections [9, 14]. Since the transmission dynamics in the Andreev-type resonator depends on g, finite frequency transport can resolve internal properties of the wire. This is, in fact, the case for the AC conductance [9, 11, 15]. However, finite frequency conductance measurements are limited in the AC frequency range since the frequency must be low enough to ensure quasi-equilibrium states in the reservoirs in order to compare experiments to existing theories. The better alternative is to apply a DC voltage and measure finite frequency current noise. Here, exploring the out of equilibrium regime, it is shown that the noise as a function of frequency has a periodic structure with period $2\pi\omega_L$, where $\omega_L = v_F/gL$ is the inverse of the traversal time of a charge excitation with plasmon velocity v_F/g through the wire of length L. The Fano factor oscillates and we will show that by averaging over $2\pi\omega_L$, the effective charge $e^* = eg$ can be extracted from noise data.

In order to analyze the noise of non-chiral LLs it is essential to study the inhomogeneous LL (ILL) model [9, 10], which takes the finite length of the interacting wire and the coupling to the reservoirs explicitly into account. This model is governed by the Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_B + \mathcal{H}_V$, where $\mathcal{H}_0$ describes the interacting wire, the leads and their mutual contacts, $\mathcal{H}_B$ accounts for the electron-impurity interaction, and $\mathcal{H}_V$ represents the coupling to the electrochemical bias applied to the wire.

Explicitly, the three parts of the Hamiltonian read

$$\mathcal{H}_0 = \frac{\hbar v_F}{2}\int_{-\infty}^{\infty} dx \left[\Pi^2 + \frac{1}{g^2(x)}(\partial_x\Phi)^2\right] , \quad (1)$$

$$\mathcal{H}_B = \lambda \cos\left[\sqrt{4\pi}\Phi(x_0,t) + 2k_F x_0\right] , \quad (2)$$

$$\mathcal{H}_V = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}}\, \mu(x)\, \partial_x\Phi(x,t) . \quad (3)$$

Here, $\Phi(x,t)$ is the standard Bose field operator in bosonization and $\Pi(x,t)$ its conjugate momentum density [16]. The Hamiltonian $\mathcal{H}_0$ describes the (spinless) ILL, which is known to capture the essential physics of a quantum wire adiabatically connected to metallic leads. The interaction parameter $g(x)$ is space-dependent and its value is 1 in the bulk of the non-interacting leads and g in the bulk of the wire ($0 < g < 1$ corresponding to repulsive interactions). The variation of $g(x)$ at the contacts from 1 to g is assumed to be smooth, i.e. to occur within a characteristic length L_s fulfilling $\lambda_F \ll L_s \ll L$, where λ_F is the electron Fermi wavelength. Since the specific form of the function $g(x)$ in the contact region will not influence physical features up to energy scales of order $\hbar v_F/L_s$, we shall, as usual, adopt a step-like function. The Hamiltonian $\mathcal{H}_B$ is the dominant $2k_F$ backscattering term at the impurity site x_0, and introduces a strong non-linearity in the field Φ. Finally, Eq. (3) contains the applied voltage. In most experiments leads are normal 2D or 3D contacts, i. e. Fermi liquids. However, since we are interested in properties of the *wire*, a detailed description of the leads would in fact be superfluous. One can account for their main effect, the applied bias voltage at the contacts, by treating them as non-interacting 1D systems ($g = 1$). The only essential properties originating from the Coulomb interaction that one needs to retain are (i) the possibility to shift the band-bottom of the leads, and (ii) electroneutrality [13]. Therefore, the function $\mu(x)$ appearing in Eq. (3), which describes the externally tunable electrochemical bias, is taken as piecewise constant $\mu(x < -L/2) = \mu_L$, $\mu(x > L/2) = \mu_R$ corresponding to an applied voltage $V = (\mu_L - \mu_R)/e$. In contrast, the QW itself does not remain electroneutral in presence of an applied voltage, and its electrostatics emerges naturally from Eqs. (1)-(3) with $\mu = 0$ for $|x| < L/2$ [11, 17].

In bosonization, the current operator is related to the Bose field Φ through $j(x,t) = -(e/\sqrt{\pi})\partial_t\Phi(x,t)$. Moreover, the finite frequency noise is defined as

$$S(x,y;\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \left\langle \{\Delta j(x,t), \Delta j(y,0)\}\right\rangle , \quad (4)$$

where $\{\,,\}$ denotes the anticommutator and $\Delta j(x,t) = j(x,t) - \langle j(x,t)\rangle$ is the current fluctuation operator. Since we investigate non-equilibrium properties of the system, the actual calculation of the averages of current and noise are performed within the Keldysh formalism [18].

The average current $I \equiv \langle j(x,t)\rangle$ can be expressed as $I = I_0 - I_{\rm BS}$, where $I_0 = (e^2/h)V$ is the current in the absence of an impurity, and $I_{\rm BS}$ is the backscattered current. For arbitrary impurity strength, temperature, and voltage, the backscattered current can be written in the compact form

$$I_{\rm BS}(x,t) = -\frac{\hbar\sqrt{\pi}}{e^2}\int_{-\infty}^{\infty} dt' \sigma_0(x,t;x_0,t') \langle j_B(x_0,t')\rangle_{\rightarrow} , \quad (5)$$

where $\sigma_0(x,t;x_0,t')$ is the non-local conductivity of the clean wire derived in [9, 11, 15]. In Eq. (5), we have introduced the "backscattered current operator"

$$j_B(x_0,t) \equiv -\frac{e}{\hbar}\frac{\delta\mathcal{H}_B}{\delta\Phi(x_0,t)}(\Phi + A_0) , \quad (6)$$

where $A_0(x_0,t)$ is a shift of the phase field emerging when one gauges away the applied voltage. For a DC voltage this shift simply reads $A_0(x_0,t) = \omega_0 t/2\sqrt{\pi}$ with $\omega_0 = eV/\hbar$ and $I_{\rm BS}$ does not depend on x and t. Furthermore, we have introduced a "shifted average" $\langle \ldots \rangle_{\rightarrow}$, which is evaluated with respect to the shifted Hamiltonian $\mathcal{H}_{\rightarrow} = \mathcal{H}_0[\Phi] + \mathcal{H}_B[\Phi + A_0]$. A straightforward though lengthy calculation shows that the finite frequency current noise (4) can (again for arbitrary impurity strength, temperature, and voltage) be written as the sum of three contributions

$$S(x,y;\omega) = S_0(x,y;\omega) + S_A(x,y;\omega) + S_C(x,y;\omega) . \quad (7)$$

The first part of Eq. (7), $S_0(x,y;\omega)$, is the current noise in the absence of a backscatterer, and can be related to the conductivity $\sigma_0(x,y;\omega)$ by the fluctuation dissipation theorem [19]

$$S_0(x,y;\omega) = 2\hbar\omega \coth\left(\frac{\hbar\omega}{2k_BT}\right) \Re[\sigma_0(x,y;\omega)] . \quad (8)$$

The conductivity can be expressed by the Kubo formula $\sigma_0(x,y;\omega) = 2(e^2/h)\omega C_0^R(x,y;\omega)$, where

$$C_0^R(x,y;\omega) = \int_0^{\infty} dt e^{i\omega t} \langle [\Phi(x,t), \Phi(y,0)]\rangle_0$$

is the time-retarded correlator of the equilibrium ILL model in the absence of an impurity. It is important to note that usually the relation (8) is only valid in thermal equilibrium, and the Kubo formula is based on linear response theory. However, due to the fact that in the absence of an impurity the current of a quantum wire attached to Fermi liquid reservoirs is linear in the applied voltage [9, 11], Eq. (8) is also valid out of equilibrium.

The other two terms in Eq. (7) arise from the partitioning of the current at the impurity site. The second term is related to the anticommutator of the backscattered current operator j_B, and reads

$$S_A(x,y;\omega) = \frac{1}{\pi}\left(\frac{h}{2e^2}\right)^2 \sigma_0(x,x_0;\omega) f_A(x_0,\omega) \sigma_0(x_0,y;-\omega) \quad (9)$$

with

$$f_A(x_0,\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \left\langle \{\Delta j_B(x_0,t), \Delta j_B(x_0,0)\}\right\rangle_{\rightarrow} ,$$

where $\Delta j_B(x,t) \equiv j_B(x,t) - \langle j_B(x,t)\rangle_{\rightarrow}$. Finally, the third part of Eq. (7) is related to the time-retarded commutator of j_B and can be expressed as

$$S_C(x,y;\omega) = \frac{h}{2e^4\omega}\Big\{S_0(x,x_0;\omega)f_C(x_0,-\omega)\sigma_0(x_0,y;-\omega) - S_0(y,x_0;-\omega)f_C(x_0,\omega)\sigma_0(x_0,x;\omega)\Big\} \quad (10)$$

with

$$f_C(x_0,\omega) = \int_0^\infty dt\,\left(e^{i\omega t}-1\right)\langle[j_B(x_0,t), j_B(x_0,0)]\rangle_{\rightarrow} \ .$$

The fractional charge is expected to emerge only in the limit of weak backscattering through the ratio between shot noise and backscattered current. We thus focus on the case of a weak impurity, retaining in the expressions (5) and (7) only contributions of second order in the impurity strength λ. Furthermore, we concentrate on the shot noise limit of large applied voltage.

The backscattering current (5) may be written as $I_{\rm BS} = (e^2/h)\mathcal{R}V$, where $\mathcal{R}$ is an effective reflection coefficient. Contrary to a non-interacting electron system, $\mathcal{R}$ depends on voltage and interaction strength [8, 20]. In the weak backscattering limit $\mathcal{R} \ll 1$, and its actual value can readily be determined from a measurement of the current voltage characteristics. Importantly, for temperatures in the window $eV\mathcal{R} \gg k_BT \gg \{\hbar\omega, \hbar\omega_L\}$ the noise can be shown to be dominated by the second term in Eq.(7) and to take the simple form

$$S(x,x;\omega) \simeq 2eF(\omega)I_{\rm BS} \ , \quad (11)$$

where $x=y$ is the point of measurement (in either of the two leads). In Eq. (11), the contributions neglected are of order $k_BT/eV\mathcal{R}$. The Fano factor

$$F(\omega) = \frac{h^2}{e^4}|\sigma_0(x,x_0;\omega)|^2 \quad (12)$$

is given in terms of the non-local conductivity $\sigma_0(x,x_0;\omega)$ relating the measurement point x to the impurity position x_0, and reads explicitly

$$F(\omega) = (1-\gamma)^2\frac{1+\gamma^2+2\gamma\cos\left(\frac{2\omega(\xi_0+1/2)}{\omega_L}\right)}{1+\gamma^4-2\gamma^2\cos\left(\frac{2\omega}{\omega_L}\right)} \ . \quad (13)$$

The latter expression is, in fact, independent of the point of measurement x and of temperature. On the other hand, it depends, apart from the frequency ω, on the (relative) impurity position $\xi_0 = x_0/L$, and the interaction strength through $\gamma = (1-g)/(1+g)$.

The central result (11) shows that the ratio between the shot noise and the backscattered current crucially depends on the frequency regime one explores. In particular, for $\omega \to 0$, the function F tends to 1, independent of the value of the interaction strength. Therefore, in the regime $\omega \ll \omega_L$ the observed charge is just the electron charge. In contrast, at frequencies comparable to ω_L the behavior of F as a function of ω strongly depends on the LL interaction parameter g, and signatures of LL physics emerge. This is shown in Fig. 1 for the case of an impurity located at the center of the wire. Then, $F(\omega)$ is periodic, and the value at the minima coincides with g^2. Importantly, g is also the mean value of F averaged over one period $2\pi\omega_L$,

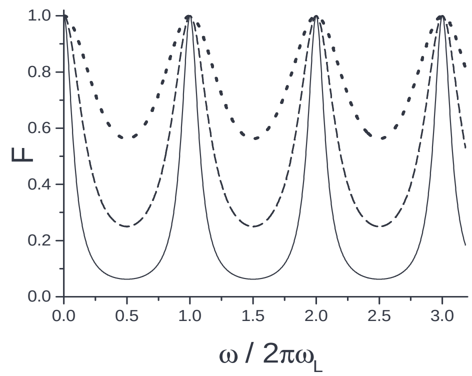

FIG. 1: The periodic function $F(\omega)$, which determines the Fano factor, is shown as a function of $\omega/2\pi\omega_L$, for the case of an impurity at the center of the wire ($x_0 = 0$) and three different values of the interaction strength: $g = 0.25$ (solid), $g = 0.50$ (dashed), and $g = 0.75$ (dotted). In the regime $\omega/\omega_L \ll 1$, the function tends to 1 independent of the value of g, but for $\omega \lesssim \omega_L$ the curve strongly depends on the interaction parameter g. In particular, g can be obtained as the average over one period.

$$\langle S(x,x;\omega)\rangle_\omega \equiv \frac{1}{2\pi\omega_L}\int_{-\pi\omega_L}^{\pi\omega_L} S(x,x;\omega) \simeq 2egI_{\rm BS} \ , \quad (14)$$

where again terms of order $k_BT/eV\mathcal{R}$ are neglected. Seemingly, Eq. (14) suggests that quasiparticles with a fractional charge $e^* = eg$ are backscattered off the impurity in the quantum wire.

Let us discuss the physical origin of this appearance of the fractional charge. We first consider the case of an infinitely long quantum wire. In the limit $L \to \infty$, i.e. $\omega_L \to 0$, $\xi_0 \to 0$, the function $F(\omega)$ becomes rapidly oscillating and its average over any finite frequency interval approaches g. Hence, we recover in this limit the result for the homogeneous LL system [3], where the shot noise is directly proportional to the fractional charge $e^* = ge$ backscattered off the impurity. However, as shown above, the value of the fractional charge e^* can be extracted not only in the borderline case $\omega \gg \omega_L$, but already for frequencies ω of order ω_L. This is due to the fact that, although the contacts are adiabatic, the mismatch between electronic excitations in the leads and in the wire inhibits the direct penetration of electrons from the leads into the wire; rather a current pulse is decomposed into a sequence of fragments by means of Andreev-type reflections at the contacts [9]. These reflections are governed by the coefficient $\gamma = (1-g)/(1+g)$, which depends on the interaction strength. The zero frequency noise is only

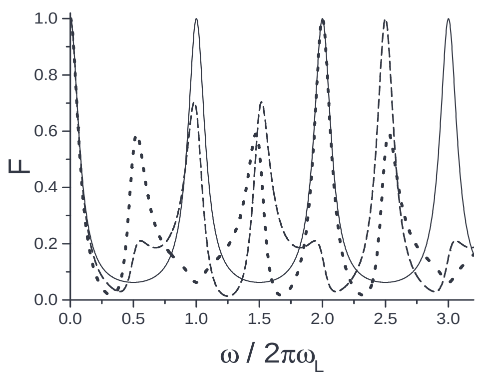

FIG. 2: The Fano factor $F(\omega)$ is shown for the interaction strength $g = 0.25$ and three different values of the (relative) impurity position $\xi_0 = x_0/L$: $\xi_0 = 0$ (solid), $\xi_0 = 0.10$ (dashed), and $\xi_0 = 0.25$ (dotted).

sensitive to the sum of all current fragments, which add up to the initial current pulse carrying the charge e. However, when $2\pi/\omega$ becomes comparable to the time needed by a plasmon to travel from the contact to the impurity site, the noise resolves the current fragmentation at the contacts. The sequence of Andreev-type processes is encoded in the non-local conductivity $\sigma_0(x, x_0; \omega)$ relating the measurement point x and the impurity position x_0. This enters into the Fano factor (12) and allows for an identification of e^* from finite frequency noise data.

When the impurity is located away from the center of the wire, $F(\omega)$ is no longer strictly periodic, as shown in Fig. 2. In that case, the combined effect of Coulomb interactions and an off-centered impurity can lead to a very pronounced reduction of the Fano factor for certain noise frequencies (see Fig. 2). Moreover, even if the impurity is off-centered, the detailed predictions (11) and (13) should allow to gain valuable information on the interaction constant g from the low frequency behavior of the Fano factor determined by

$$F(\omega) = 1 - (1 - g^2)(1 + 4g^2\xi_0(1 + \xi_0)) \left(\frac{\omega L}{2v_F}\right)^2 + \dots .$$

The latter expression is valid in the parameter regime $eV\mathcal{R} \gg k_B T \gg \hbar\omega_L \gg \hbar\omega$.

In conclusion, the appearance of fractional charge $e^* = eg$ in the finite frequency noise of non-chiral LLs is due to a combined effect of backscattering of bulk quasiparticles at the impurity and of Andreev-type reflections of plasmons at the interfaces of wire and leads. The fractional charge e^* can be extracted from the noise by averaging it over a frequency range $[-\pi\omega_L, \pi\omega_L]$ in the out of equilibrium regime. For single-wall carbon nanotubes we know that $g \approx 0.25$, $v_F \approx 10^5$ m/s, and their length can be up to 10 microns. Thus, we estimate $\pi\omega_L \approx 100$ GHz $\dots 1$ THz, which is a frequency range that seems to be experimentally accessible [21, 22]. Moreover, the requirement $eV \gg \hbar\omega_L$ should be fulfilled in such systems for $eV \approx 10 \dots 50$ meV, a value which is well below the subband energy separation of about 1 eV.

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